

Level - 1	JEE Mains Pattern/Solutions
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76.(2) $\ell = 1m, A = 2 \times 10^{-6} m^2$

$$F = 20N, \Delta \ell = 0.5 \times 10^{-4} m$$

$$Y = \frac{F/A}{\Delta \ell / \ell} = \frac{20}{2 \times 10^{-6}} \times \frac{1}{0.5 \times 10^{-4}} = \frac{20}{1 \times 10^{-10}}$$

$$= 20 \times 10^{10} = 2 \times 10^{11} N/m^2$$

$$\text{In } 10^{11} N/m^2, Y = 2.$$

77.(2) $mg = (K) (9) \text{ or } K = mg/9$

The spring constant is inversely proportional to length. So the spring constant for the spring of 40 cm length is $\frac{3}{2}K$. The spring constant for the spring of 20 cm length is 3 K. The spring constant for the combination is

$$\frac{3}{2}K + 3K = \frac{9}{2}K; \text{ Now } mg = \frac{9}{2}Kx_0 \text{ or } 9K = \frac{9}{2}Kx_0 \quad x_0 = 2cm$$

78.(4) Breaking strength = tension = $mr\omega^2$

$$4.8 \times 10^7 \times 10^{-6} = 10 \times 0.3 \omega^2; \quad \omega = 4 rad/sec.$$

79.(4) $\frac{\theta_1 - \theta_2}{t} = \alpha \left(\frac{\theta_1 + \theta_2}{2} - \theta_0 \right)$

$$\text{In the first case, } \frac{75 - 65}{2} = \alpha \left[\frac{75 + 65}{2} - 30 \right] \quad \dots(i)$$

$$\text{In second case, } \frac{55 - 45}{t} = \alpha \left[\frac{55 + 45}{2} - 30 \right] \quad \dots(ii) \quad \text{Solving } \Rightarrow t = 4 \text{ min}$$

80.(8) Rate of conduction $R \propto \frac{r^2}{\ell}$. The ratio of conduction in them is $\frac{r^2}{\ell} : \frac{r^2}{2\ell} : \frac{2r^2}{\ell} : \frac{4r^2}{\ell}$, i.e., 2 : 1 : 4 : 8

So the ratio of maximum and minimum conduction rate is 8 : 1, i.e., 8.

81.(2) Loss or gain per day = $dT = \frac{1}{2} \alpha dt \times 86400$

Since, $T = 86400$ s for each day

$$\text{At } 15^\circ\text{C, } 5 = \frac{1}{2} \alpha (t - 15)(86400); \text{ At}$$

$$30^\circ\text{C, } 10 = \frac{1}{2} \alpha (30 - t) \times 86400$$

$$\Rightarrow \frac{30 - t}{t - 15} = 2 \Rightarrow t = 20^\circ\text{C s}$$

82.(9) $(\lambda_m)_B = 3(\lambda_m)_A \Rightarrow T_A = 3T_B$

$$E_1 = 4\pi(6)^2 \sigma T_A^4 = 4\pi(6)^2 (3T_B)^4$$

$$E_2 = 4\pi(18)^2 \sigma T_B^4, \quad \frac{E_1}{E_2} = 9$$

83.(9) Increase in pressure is $\Delta p = \frac{Mg}{A}$

$$\text{Bulk modulus is } B = \frac{\Delta p}{(\Delta V / V)}$$

$$\therefore \frac{\Delta V}{V} = \frac{\Delta p}{B} = \frac{Mg}{AB} \quad \dots (i)$$

Also, the volume of the sphere is

$$V = \frac{4}{3}\pi R^3 \Rightarrow \frac{\Delta V}{V} = \frac{3\Delta R}{R}$$

or $\frac{\Delta R}{R} = \frac{1}{3} \cdot \frac{\Delta V}{V}$, using eq. (i), we get

$$\frac{\Delta R}{R} = \frac{Mg}{3AB} \quad \therefore n = 9$$

84.(2) From $T = 2\pi\sqrt{\frac{\ell(1+\alpha T)}{g}}$

We get, $\frac{dT}{T} = \frac{1}{2}\alpha dt$

Loss or gain per day $= dT = \frac{1}{2}\alpha dt \times 86400$

Since $T = 86400$ s for each day,

At 15°C , $5 = \frac{1}{2}\alpha(t-15) \times 86400$; At 30°C , $10 = \frac{1}{2}\alpha(30-t) \times 86400$

$$\therefore \frac{30-t}{t-15} = 2 \Rightarrow 3t = 60^\circ\text{C}; T = 20^\circ\text{C}$$

$$\therefore \alpha = \frac{10}{(t-15) \times 86400} = \frac{10}{5 \times 86400} = 0.0000023 = 2.3 \times 10^{-6}/^\circ\text{C} = 2$$

85.(6) Energy with 5 kg of H_2O of 20°C to become ice at 0°C $E_1 = 5000 \times 1 \times 20 = 100000 \text{ cal}$

Energy to raise the temperature of 2 kg ice from -20°C to 0°C is $E_2 = 2000 \times 0.5 \times 20 = 20000 \text{ cal}$.

$(E_1 - E_2) = 80,000 \text{ cal}$ is available to melt ice at 0°C . So only 1000 g or 1 g of ice would have melt.
So amount of water available is $1+5=6$.

86.(4) From $\frac{\theta_1 - \theta_2}{t} = \alpha \left(\frac{\theta_1 + \theta_2}{2} - \theta_0 \right)$, in first case we have,

$$\frac{75-65}{2} = \alpha \left[\frac{75+65}{2} - 30 \right] \quad \dots\dots(i)$$

For second case,

$$\frac{55-45}{2} = \alpha \left[\frac{55+45}{2} - 30 \right] \quad \dots\dots(ii)$$

Dividing (i) by (ii), $\frac{t}{2} = 2$

$T = 4 \text{ min}$

87.(4) $\frac{\text{Temperature difference}}{\text{Thermal resistance}} = L \left(\frac{dm}{dt} \right)$

$$\frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}}; \quad q \propto \frac{1}{R}$$

The rods are in parallel in the first case and they are in series in the second case. $\frac{q_1}{q_2} = \frac{2R}{(R/2)} = 4$

88.(4) Area of top surface is $A = 3 \text{ cm} \times 1 \text{ cm} = 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2$

Shearing stress, $\sigma = \frac{F}{A} = \frac{2.1}{3 \times 10^{-4}} = 0.7 \times 10^4 \text{ Nm}^{-2}$

Angle of shear, $\theta = \frac{\text{Shearing stress}}{\text{Shear modulus}} = \frac{0.7 \times 10^4}{1.4 \times 10^5} = 0.05 \text{ rad}$

89.(2) Length of the second rod at 25°C is 70 cm .

Length of the copper rod at 125°C is $30 \times (1 + 1.7 \times 10^{-5} \times 100) = 30.051\text{ cm}$.

Increase in the length of second rod is $70 \times \beta \times 100 = 7000 \beta\text{ cm}$

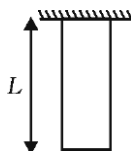
Total increase in length $= 0.051\text{ cm} + 7000 \beta\text{ cm} = 0.191\text{ cm}$ (given), which given $\beta = 2 \times 10^{-5}\text{ per }^{\circ}\text{C}$.

90.(6) If L is the maximum length of the wire that can be suspended. Then $F = mg$, where $m = \rho LA$

(where, A = cross-sectional area)

$$\therefore \text{Maximum stress} = \frac{mg}{A}$$

$$= g \times \frac{\rho LA}{A} = \rho Lg$$



$$\text{Given, breaking stress} = \sigma_b = 9 \times 10^{10} \text{ Nm}^{-2} \quad \therefore \quad \rho Lg = 9 \times 10^{10}$$